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SUBJECT: Apollo Communications and Tracking System Performance During Earth Parking Orbit Phase - Case 20061

DATE: February 17, 1964

FROM: R. W. Priester
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20 p.

MEMORANDUM FOR FILE

The effect of radar data and site location errors on orbit determination during the earth parking orbit phase has been investigated. This memorandum reports the results of the investigation for the case of several stations tracking during one complete earth orbit. Results were reported earlier[1] for a single ship tracking immediately after insertion into earth parking orbit.

The method used was a computer simulation and is described in detail in the report of the previous investigation. Basically, two computer programs were involved. The first simulated a radar by generating radar data (azimuth, elevation and range) corresponding to discrete times during the pass of an orbiting satellite over a tracking station. The second program was used to compute the orbit of the satellite, given the radar data and the location of the tracking station. Errors were deliberately introduced into the data to simulate random and bias errors in the radar and uncertainties in site location. By comparing the computed orbit with the one used to generate the radar data, the effect of the errors was observed.

The tracking sites considered for this investigation were one ship in the Atlantic Ocean and the following land stations:

Bermuda	Guam
Antigua	Hawaii
Ascension	Pt. Arguello
Carnarvon	Cape Kennedy

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[1] J. O. Replogle, "Apollo C&TS - Analysis of Tracking Ship Performance" Case 20053-1, MFF dated 10/16/63.

(NASA-CR-126777) APOLLO COMMUNICATIONS AND TRACKING SYSTEM PERFORMANCE DURING EARTH PARKING (Bell Telephone Labs., Inc.) 20 p

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A single position for the Atlantic ship would not permit adequate tracking for all launch azimuths within the permissible range of 72° to 108°. The range of launch azimuths that might be used on any one day, however, is expected to be only about 26°, which can be covered adequately from a single position of the ship; furthermore, the midpoint of the most favorable 26° range moves slowly from day to day, so that a ship could move to maintain a favorable position. For this investigation two positions were chosen for the ship, each to permit acquisition of the spacecraft shortly before insertion into earth parking orbit. One position was chosen to give equal coverage for launch azimuths of 72° and 102°, and the other to give equal coverage for 78° and 108°. Thus, from either position the ship could cover all but 6 degrees of the 72°-108° range. The two positions and the resulting coverage of several tracks are shown in Figure 1.

The tracking coverage provided by the ship and land stations is shown in Figure 2. It is evident that the coverage improves with increasing (more southerly) launch azimuths within the range considered, i.e., the coverage is poorest for 72° and best for 108°. These two extremes of launch azimuth were selected for further analysis.

A radar data sampling interval of six seconds was assumed. Data were used only above 5 degrees elevation, and only after insertion into orbit. For the 72° launch azimuth, these restrictions limited the radar coverage to that provided by the ship, Carnarvon and Cape Kennedy. For the 108° launch azimuth, all eight stations shown in Figure 2 were used. The tracking times for the sites are summarized in Table 1.

The random errors applied to the radar data were from normal distributions having standard deviations of 0.3 milliradians in angle and 50 feet in range. A different sample from the appropriate distribution was applied to each value of range and angle.

Bias errors were also drawn from normal distributions, but the same value of bias was applied to all measurements of a given coordinate at a given site on a given pass. Different sites and passes had different biases, however. The standard deviations of noise and bias errors are given in Table 2.

The sources of bias errors in a shipboard radar are discussed in the preceding report[1]. The biases for land stations shown

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[1]ibid

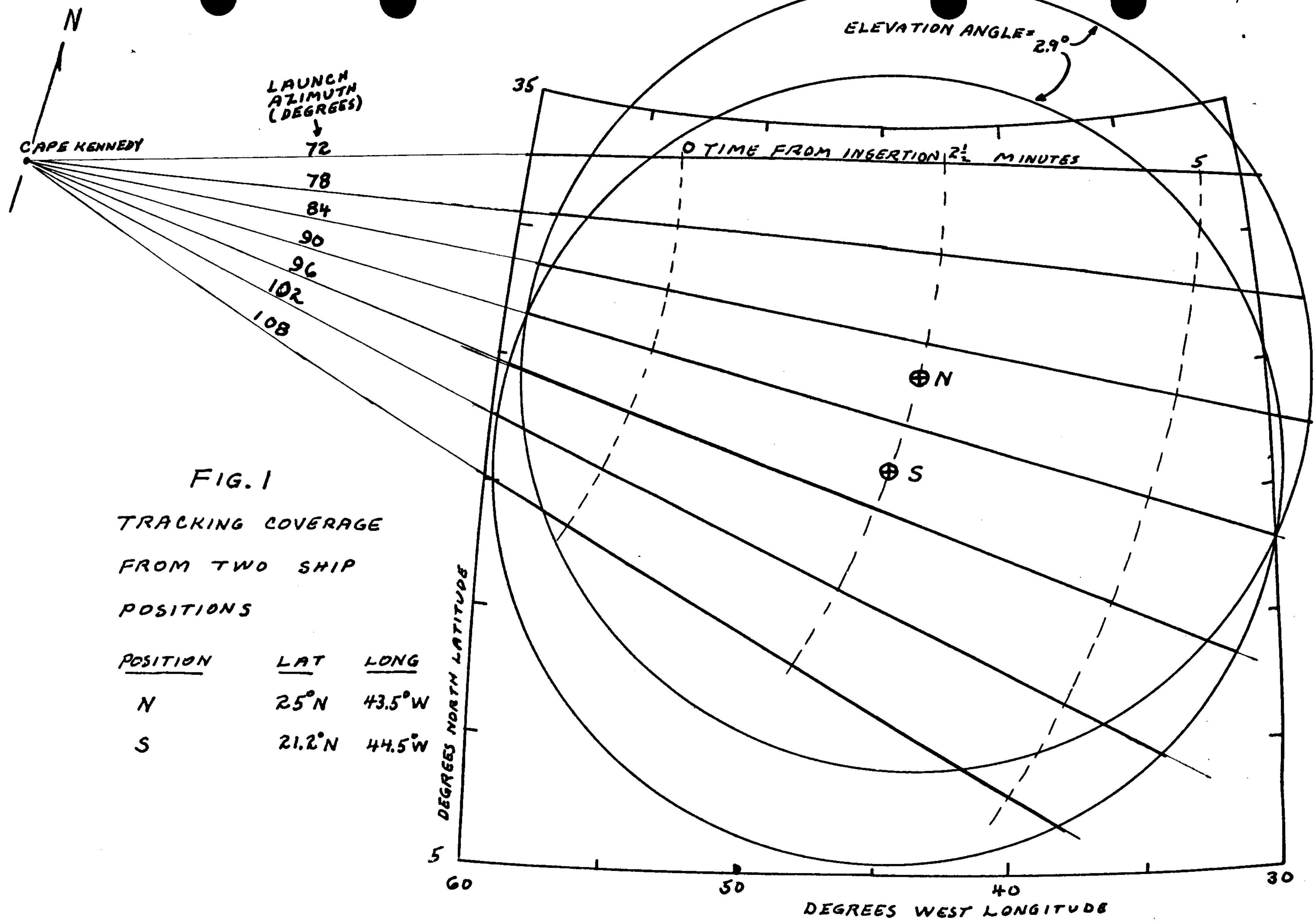

FIG. 1

TRACKING COVERAGE FROM TWO SHIP POSITIONS

POSITION	LAT	LONG
N	25°N	43.5°W
S	21.2°N	44.5°W

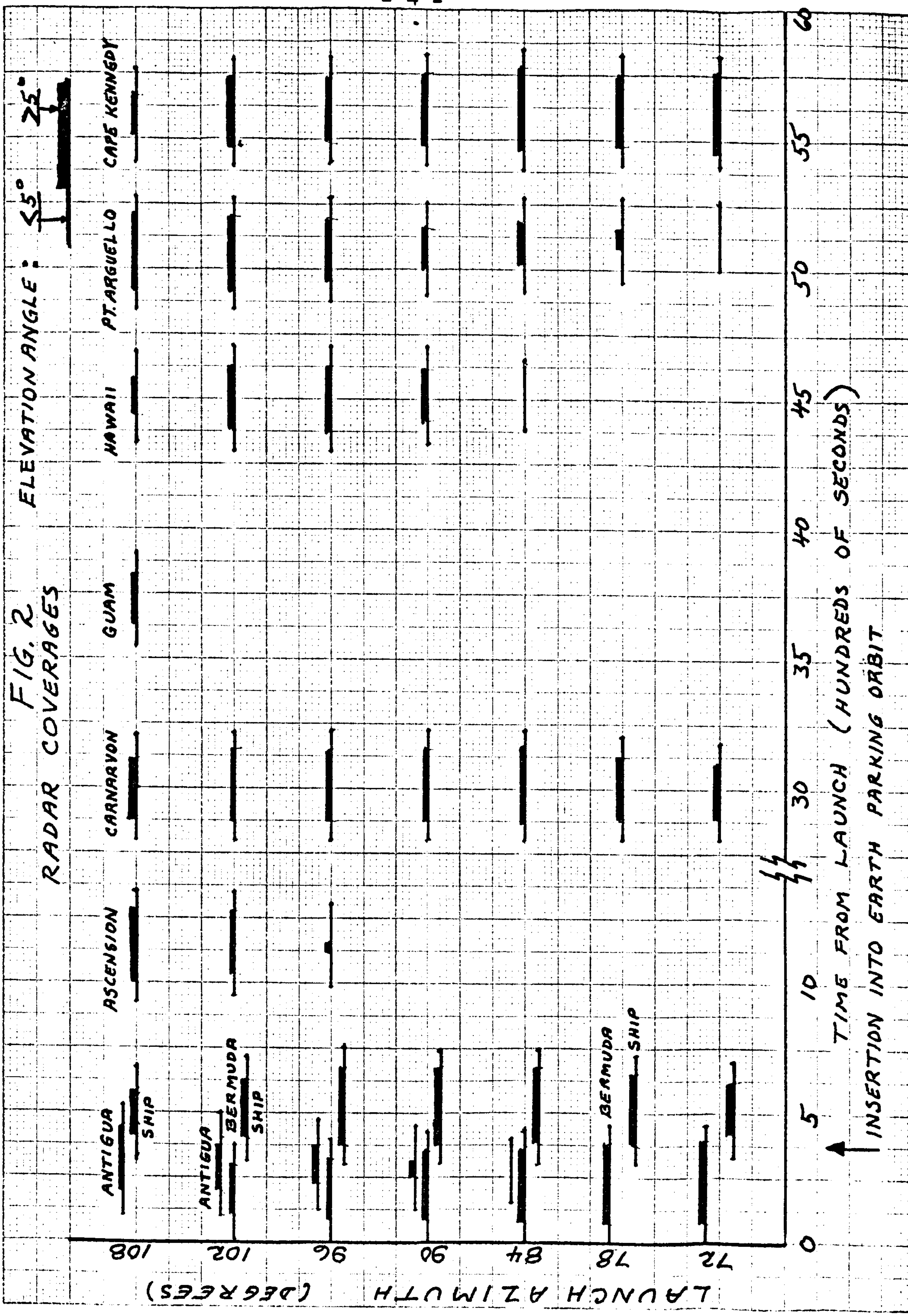

FIG. 2
RADAR COVERAGES

TABLE 1

TRACKING TIMES*

	72°	108°
Antigua		1-1/2 minutes
Ship	3	3
Ascension		5
Carnarvon	3	4
Guam		3
Hawaii		3
Pt. Arguello		5
Cape Kennedy	5	3-1/2
Total	11	28 minutes

*Above 5° elevation

TABLE 2

SOURCE OF ERROR	STANDARD DEVIATION
Radar	
Noise	
Azimuth	0.3 mils
Elevation	0.3 mils
Range	50 feet
Bias	
Azimuth	0.08 (0.17*) mils
Elevation	0.10 (0.11*) mils
Range	10 feet
Site	
Latitude	60 (1500*) feet
Longitude	60 (1500*) feet
Altitude	85 feet

*For ship

in the table are the same except for those sources of error which are peculiar to shipboard installations (e.g., inertial reference and navigation errors).

The computer simulation technique is illustrated in Figure 3. ORACLE is an orbit refinement program and requires as inputs, in addition to radar data and site coordinates, an estimate of the orbit in terms of six orbital elements and the inverse of their covariance matrix. The orbital elements and their values are:

ORBITAL ELEMENTS

	Launch Azimuth 72°	Launch Azimuth 108°	
node angle	-.97	-2.18	radians
inclination	.58	.58	radians
period	5310	5310	seconds
e cos ω*	0	0	
e sin ω*	0	0	
time of ascending node	-890	-1765	seconds

*e = eccentricity

*ω = argument of perigee

For the first site the true values of the orbital elements are used and the covariance matrix is set equal to zero, so that the true values are given no weighting in determining the output set of elements. Random and bias errors are added to the radar data and bias errors are added to the site coordinates before they are used in the computations. The output of ORACLE is a set of orbital elements based on the perturbed data, along with its associated inverse covariance matrix. These outputs are then supplied as inputs to the program along with data from the next site. The program in this way effectively combines the data from two sites to obtain the next set of orbital elements. This process is continued until the data from all sites has been assimilated. The entire foregoing procedure is repeated some 25 times using different samples of random numbers for the random and bias errors, to obtain results with statistical significance.

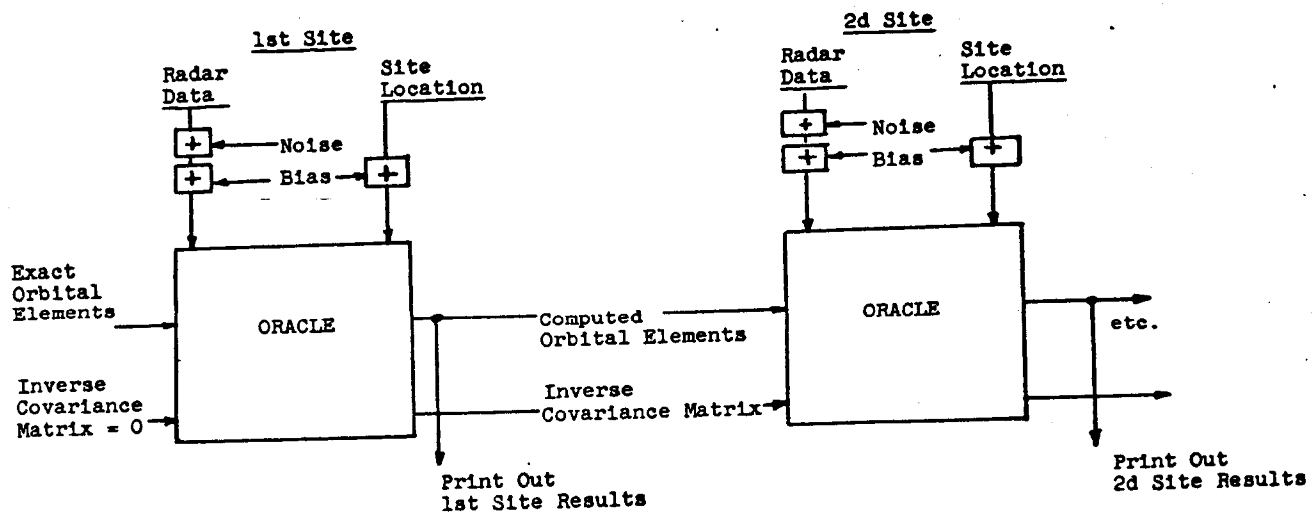

FIG. 3

Computation Technique

Another input to the ORACLE program, not shown in Figure 3, is an estimate of the variance of the radar data. This information is used to give appropriate weighting to the range, azimuth and elevation data reflecting their relative accuracies. The weighting scheme was designed to take account of random errors in the radar data; however, in the situation being simulated here, additional uncertainties in the radar measurements are introduced by bias errors--not only in the radar itself, but also (and to a greater degree) in the location of the radar. There are a number of ways that biases might be taken into account in weighting the radar data. One might reason that the data should be weighted according to the mean square error in each coordinate (range, azimuth and elevation), i.e., use as the variance the sum of random and bias errors. This would still leave the question of how to treat bias errors in site location.

These site biases, in the local horizontal and vertical, might be resolved into components corresponding to azimuth, elevation and range, and then treated as bias errors in radar coordinates. This procedure would be rather unwieldy; moreover, considering the relative magnitude of range and angle errors and the geometry of the situation, the angle errors would not be greatly affected by the addition of site errors. The range errors, however, would be affected to a considerable degree, particularly in the case of the ship, which was assumed to have a standard deviation of error of 1500 feet in the local horizontal. Since most of the tracking data is obtained at elevation angles of less than 45 degrees, a large component of the 1500-foot position error would appear in range. In the case of a land station, the uncertainties of position in the local horizontal (60 ft.) and in the local vertical (85 ft.) are so nearly equal that the apparent range error due to errors in site location is substantially independent of the elevation angle. Thus, adding the variances of horizontal site location and range bias, then treating the sum as the variance of range bias, (and neglecting the effect of site bias on angle measurements) should approximate reasonably well the transformation of site biases into radar coordinates.

A significant difference between random errors and biases in measurements from a single site is that the effect of random errors can be reduced by taking a number of observations, while bias errors are not amenable to smoothing. Since smoothing over n observations reduces the random errors by a factor of $\sqrt{n}$ without affecting the bias errors, it would seem appropriate to assume a standard deviation in the case of bias $\sqrt{n}$ times the actual bias error.

It is evident that the subject of the treatment of bias errors deserves further study. A rigorous analysis was not attempted in the investigation, but several different weighting schemes were used. While the results do not clearly establish an optimum method of weighting, they demonstrate that the accuracy of orbit determination depends on not only the accuracy of the radar data, but on the relative weight assigned to the data as well.

The results of three different ways of weighting the data are presented. The methods of weighting are:

1. Biases are neglected. Data are weighted according to the variances of random error in the three radar coordinates (standard deviations of 0.3 mil in angle and 50 feet in range).

2. An approximate correction for site bias is made by increasing the assumed standard deviation of range (for the purpose of weighting) to 100 feet for a land station and 2000 feet for a ship. Angles (azimuth and elevation) are weighted as in (1).

3. The effect of smoothing is taken into account, and the effects of bias and random errors are combined, by assuming the variance for weighting the data (azimuth, elevation or range) to be:

   $$\sigma^2 = \sigma_N^2 + n\sigma_B^2$$

   where σ_N = standard deviation of random error (Noise) in a single observation

   σ_B = standard deviation of bias error
   n = number of observations from the site.

   For azimuth and elevation, σ_B was the actual standard deviation of bias error in the radar coordinates. For range, σ_B was the root-sum square of range bias and horizontal site bias (latitude or longitude).

The results of these simulations are expressed in terms of the standard deviation of error (based on about 25 trials) in a number of parameters, including the six orbital elements defined previously, and position and velocity in the tangential, radial, and normal directions at the tracking site. Some results are presented below. Others are given in the Appendix.

It is important to know, as soon as possible after the end of powered flight, whether at least one complete earth orbit can be achieved. This can be indicated by the height of perigee. Table 3 shows errors in the computed height of perigee for the three data-weighting methods described above, and for launch azimuths of 72 and 108 degrees. While the estimate of height of perigee from the first tracking site is of primary importance, the changes in this initial estimate with data from succeeding tracking stations are also shown.

While the accuracy of the calculations varies with the site configuration and the method of weighting the data, these results indicate that the height of perigee can be established to within a mile or two after tracking by the first site after insertion. It will be noted that for the 108° launch azimuth and for the first two weighting methods, using data from the ship degrades the accuracy of the calculation obtained with data from Antigua. This is probably the result of the method of combining data of vastly different accuracies very early in the orbit determination (i.e., before the orbit has been firmly established by a long period of previous tracking). It should be pointed out that the computational methods used in the ORACLE program were designed for using tracking data to further refine an established orbit, and these methods are not optimum for the initial determination of an orbit.

While the height of perigee is probably the most critical parameter immediately after insertion, injection into lunar transfer trajectory requires accurate knowledge of the spacecraft position and velocity immediately after injection burn. Position and velocity errors have been calculated at a point in the vicinity of each site, using data from that site (as well as previous sites). Standard deviations of error in three orthogonal directions were calculated and are included in the Appendix. These are summarized in Table 4 in the form of rms position and velocity errors. Of the three weighting methods used, the first (using the variance of the random errors) resulted in the smallest errors for the 72° launch azimuth, 3-site case. In the 108-degree case, the first method resulted in the smallest errors at the first site, but the other two methods worked better at succeeding sites.

TABLE 3

ERRORS IN HEIGHT OF PERIGEE
(standard deviations, feet)

Launch Azimuth	Site	Data weighting method* 1	2	3
72°				
	Ship	2588	5962	6038
	Carnarvon	288	283	285
	Cape Kennedy	180	165	155
108°				
	Antigua	4112	4400	5987
	Ship	6828	8961	7822
	Ascension	3175	278	259
	Carnarvon	159	72	78
	Guam	137	31	35
	Hawaii	61	29	30
	Pt. Arguello	32	20	17
	Cape Kennedy	47	21	23

*described in text

TABLE 4

R.M.S. POSITION AND VELOCITY ERRORS*

Launch Azimuth	Site	Data Weighting Method 1	2	3
72°				
	1	2710 (6.72)	12,500 (7.85)	12,500 (8.46)
	2	414 (0.94)	682 (2.13)	820 (2.66)
	3	216 (0.57)	200 (1.41)	225 (1.71)
108°				
	1	7970 (5.81)	7990 (5.81)	9090 (6.11)
	2	9720 (4.85)	8430 (5.29)	7850 (4.68)
	3	1670 (1.95)	389 (0.30)	406 (0.29)
	4	427 (0.41)	174 (0.15)	182 (0.16)
	5	585 (0.40)	123 (0.10)	187 (0.12)
	6	415 (0.37)	138 (0.08)	141 (0.07)
	7	238 (0.23)	126 (0.06)	111 (0.05)
	8	243 (0.11)	132 (0.05)	123 (0.05)

*position, feet
(velocity, feet per second)

Any comparison of the several weighting methods should be made in the light of the small sample size on which the results are based. An estimate of the standard deviation of a population based on a sample of 25 (the approximate number of runs made under each set of conditions) has 90 per cent confidence limits* of approximately .8 and 1.3 times the calculated value. The differences between weighting methods seem less significant when these limits are considered. As an example, Table 5 gives confidence limits on the error in height of perigee for the first two tracking sites on the 108° launch azimuth.

TABLE 5

90 Per Cent Confidence Limits on Height of Perigee
(Standard Deviation of Error, Feet)

	Weighting Method		
	1	2	3
Antigua	3300-5400	3500-5800	4800-7900
Ship	5500-9000	7200-11,800	6300-10,300

The results of this investigation indicate the kind of accuracies that can be achieved with an existing computer program. They also emphasize the difficulties associated with combining data from different sources of different accuracies. In particular, the problem of using data from a ship whose location is uncertain is emphasized. The need for further study in (1) methods of using data with large errors (particularly biases) and (2) possible ways of reducing the uncertainty of a tracking ship's position is indicated.

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Att.
Appendix

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*According to a method by Blackman and Tukey

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TABLE A-1

LAUNCH AZIMUTH 72 DEGREES

SITE	NØDE ANGLE RADIANS	INCLNATN ANGLE RADIANS	PERIØD SECØNDS	ECCENT. X CØSINE ØMEGA	ECCENT. X SINE ØMEGA	TIME ØF NØDE	HT. ØF PER. FEET
WEIGHTING 1							
SHIP	0.000310	0.000013	0.450	0.000181	0.000042	0.376	2588.0
CARN	0.000050	0.000003	0.062	0.000022	0.000007	0.045	288.1
C.KN	0.000034	0.000001	0.066	0.000010	0.000002	0.043	179.6
WEIGHTING 2							
SHIP	0.000324	0.000013	2.268	0.000184	0.000286	0.719	5962.0
CARN	0.000136	0.000008	0.071	0.000020	0.000009	0.096	282.9
C.KN	0.000092	0.000008	0.055	0.000008	0.000003	0.096	165.1
WEIGHTING 3							
SHIP	0.000348	0.000014	2.277	0.000185	0.000287	0.739	6038.0
CARN	0.000176	0.000011	0.073	0.000019	0.000009	0.121	285.2
C.KN	0.000114	0.000009	0.053	0.000008	0.000004	0.104	154.7

TABLE A-2

LAUNCH AZIMUTH 108 DEGREES

SITE	NØDE ANGLE RADIANS	INCLNATN ANGLE RADIANS	PERIØD SECØNDS	ECCENT. X CØSINE ØMEGA	ECCENT. X SINE ØMEGA	TIME ØF NØDE	HT. ØF PER. FEET
WEIGHTING 1							
ANTG	0.000158	0.000108	1.134	0.000134	0.000182	0.467	4112.0
SHIP	0.000063	0.000014	2.267	0.000255	0.000206	0.855	6828.0
ASCN	0.000003	0.000051	1.014	0.000117	0.000031	0.547	3175.0
CARN	0.000029	0.000005	0.045	0.000008	0.000006	0.044	158.9
GUAM	0.000015	0.000010	0.044	0.000005	0.000008	0.040	137.4
HWAI	0.000016	0.000008	0.015	0.000002	0.000004	0.021	61.2
PT.A	0.000016	0.000005	0.010	0.000002	0.000001	0.006	32.4
C.KN	0.000014	0.000003	0.012	0.000003	0.000001	0.004	47.2
WEIGHTING 2							
ANTG	0.000157	0.000107	1.209	0.000140	0.000183	0.491	4400.0
SHIP	0.000098	0.000070	2.887	0.000322	0.000184	1.182	8961.0
ASCN	0.000003	0.000004	0.106	0.000012	0.000008	0.051	278.0
CARN	0.000004	0.000002	0.027	0.000004	0.000002	0.016	71.9
GUAM	0.000002	0.000003	0.012	0.000002	0.000002	0.010	31.1
HWAI	0.000002	0.000001	0.011	0.000001	0.000002	0.009	29.3
PT.A	0.000002	0.000001	0.006	0.000001	0.000001	0.006	19.9
C.KN	0.000002	0.000001	0.005	0.000001	0.000001	0.005	21.4
WEIGHTING 3							
ANTG	0.000154	0.000104	1.706	0.000184	0.000210	0.679	5987.0
SHIP	0.000095	0.000065	2.456	0.000267	0.000171	1.008	7822.0
ASCN	0.000003	0.000004	0.094	0.000011	0.000009	0.049	259.1
CARN	0.000003	0.000002	0.031	0.000004	0.000003	0.019	78.2
GUAM	0.000002	0.000003	0.015	0.000002	0.000002	0.011	34.9
HWAI	0.000002	0.000001	0.010	0.000001	0.000002	0.008	30.5
PT.A	0.000002	0.000001	0.005	0.000001	0.000001	0.005	17.1
C.KN	0.000002	0.000001	0.005	0.000001	0.000001	0.005	22.8

TABLE A-3

LAUNCH AZIMUTH 72 DEGREES

SITE	PØSITIØN ERRØRS TANG FT.	RADIAL FT.	NØRMAL FT.	ANGLE ERRØRS HEADING RAD.	FLTPATH RAD.	VELØCITY ERRØRS TANG FT/SEC	NØRMAL FT/SEC	RADIAL FT/SEC
WEIGHTING 1								
SHIP	2744.	280.	191.	0.000190	0.00018C	0.52	4.85	4.59
CARN	323.	201.	178.	0.000030	0.000021	0.20	0.76	0.54
C.KN	112.	161.	97.	0.000020	0.00001C	0.10	0.51	0.25
WEIGHTING 2								
SHIP	12506.	281.	228.	0.000199	0.000189	3.60	5.07	4.81
CARN	395.	180.	525.	0.000081	0.000019	0.20	2.07	0.49
C.KN	128.	108.	109.	0.C00055	0.000008	0.C7	1.40	0.20
WEIGHTING 3								
SHIP	12529.	281.	243.	0.000213	0.000189	3.62	5.44	4.82
CARN	436.	180.	689.	0.000104	0.000019	0.21	2.66	0.48
C.KN	149.	112.	135.	C.000068	0.000008	0.C9	1.73	0.19

TABLE A-4

LAUNCH AZIMUTH 108 DEGREES

SITE	PØSITIØN ERRØRS TANG FT.	RADIAL FT.	NØRMAL FT.	ANGLE ERRØRS HEADING RAD.	FLTPATH RAD.	VELØCITY ERRØRS TANG FT/SEC	NØRMAL FT/SEC	RADIAL FT/SEC
WEIGHTING 1								
ANTG	8127.	138.	540.	0.000133	0.000175	1.85	3.40	4.45
SHIP	9905.	759.	412.	0.000024	0.0C0078	4.50	0.61	2.00
ASCN	1616.	172.	509.	0.C00052	0.00000C9	1.46	1.34	0.22
CARN	21C.	18C.	337.	0.000012	0.00001C	0.14	0.30	0.25
GUAM	536.	254.	28.	0.000012	0.000006	0.23	0.30	0.16
HWAI	397.	137.	42.	0.C00014	0.000001	0.14	0.35	0.04
PT.A	143.	41.	182.	C.C00009	0.000002	0.C4	0.22	0.06
C.KN	109.	81.	206.	0.C00002	0.000002	0.C8	0.C5	0.05
WEIGHTING 2								
ANTG	8150.	135.	537.	0.C00132	0.000173	1.96	3.36	4.40
SHIP	8476.	223.	233.	0.000082	0.0C0049	4.74	2.10	1.26
ASCN	357.	104.	45.	C.000004	0.000009	0.16	0.11	0.23
CARN	122.	109.	68.	C.C00003	0.000004	0.09	0.07	0.10
GUAM	103.	44.	19.	C.C00003	0.0C0002	0.C4	C.07	0.05
HWAI	127.	52.	21.	C.C00002	0.000002	0.C5	C.04	0.05
PT.A	109.	46.	29.	C.C00001	0.000001	0.04	0.03	0.03
C.KN	123.	44.	30.	C.C00001	0.000001	0.C4	0.02	0.02
WEIGHTING 3								
ANTG	9275.	134.	510.	C.C00129	0.000178	2.75	3.28	4.53
SHIP	7962.	258.	190.	C.000077	0.0C0066	3.99	1.97	1.68
ASCN	384.	101.	40.	0.C00004	0.000009	0.16	0.10	0.22
CARN	131.	120.	51.	0.000002	0.000005	0.10	C.06	0.12
GUAM	146.	76.	19.	0.000003	0.000002	0.C7	0.07	0.05
HWAI	127.	54.	22.	C.C00001	0.000001	0.C5	0.03	0.04
PT.A	90.	38.	25.	0.000001	0.000001	0.C4	0.03	0.02
C.KN	114.	40.	29.	C.C00001	0.000001	0.C4	0.02	0.02